Identification of Parallel Flows in Power Networks through State Estimation and Hypothesis Testing *

Kevin A. Clements\textsuperscript{a}, Antonio Simões Costa\textsuperscript{b,*},
Angela Agudelo\textsuperscript{b}

\textsuperscript{a}Worcester Polytechnic Institute, Worcester, MA 01609, USA
\textsuperscript{b}Electrical Engg. Dept., Universidade Federal de Santa Catarina, 88040-900 Florianópolis, SC, Brazil

Abstract

The occurrence of multiple bilateral transactions in large interconnected power networks may expose a given power system control area to large unexpected parallel flows that are usually detrimental to the area economic operation, in addition to posing security risks. To prevent undesirable consequences as well as to take adequate compensation measures, the control area operator must be able to identify the causes of such unaccounted power flows.

The purpose of this paper is to present a monitoring tool to track parallel flows due to power transactions unreported to the control area operator. The proposed tool extends the role of Power System State Estimation through the definition of additional state variables representing bilateral transactions, which are then estimated along with the conventional states. The unreported transactions are identified through statistical tests based on Bayes’s theorem. The paper also discusses relevant computational issues related to the method’s implementation. The IEEE 118-bus test system is employed to evaluate the performance of the proposed monitoring tool.

Key words: Power System Real-Time Modeling, Power System State Estimation, Operation of Deregulated Power Systems

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* Corresponding author.
1 Introduction

Deregulation of the power industry has created favorable conditions to the establishment of bilateral contract markets under which bilateral energy transactions between generating companies and electric energy buyers take place. In deregulated environments, the Independent System Operator (ISO) has the task of monitoring the electric network in order to ensure the secure operation of the power system. To accomplish that, the ISO must be aware of all bilateral transactions within its control area. However, bilateral transactions occurring between entities external to the ISO’s control area can also affect power flows within it [1]. Although in normal operating situations external bilateral transactions are disclosed to the ISO, in practice this may not always be the case. Inadvertent events such as unforeseen changes in the network topology and other causes [1] may expose a given control area to large unexpected parallel flows. As a consequence, increased network losses and transmission congestion may result, with detrimental impacts to the system’s economic and secure operation [1],[2].

The need then arises for reliable monitoring tools to assist the system operator in identifying the causes of unaccounted flows, so that adequate compensation may be made. The ISO’s basic system monitoring tool is Power System State Estimation (PSSE). It is well known that PSEE can handle modeling errors due to bad data both in analog (see [3], for instance) and digital measurements, the latter usually leading to topology errors [4],[5]. However, conventional PSSE is not well suited to identify unreported transactions, as they manifest themselves via unexpected power injections at boundary buses which not necessarily produce inconsistencies within the internal system. In addition, in deregulated environments the number of bilateral transactions that may be simultaneously taking place in an interconnected power system may be quite large [1] and they usually involve several boundary buses of a given control area. It is thus a difficult task to trace unreported transactions by simply looking at deviations of tie line power flows from expected values.

This paper extends conventional state estimation in order to allow the identification of unreported transactions. The proposed PSSE extension exhibits the following characteristics: (a) The bilateral transactions which the ISO’s control area is exposed to are treated as new state variables to be estimated along with the conventional states; and (b) Unreported transaction identification is formulated as a hypothesis testing problem whose solution is based on Bayesian statistics. In addition to establishing the theoretical framework, this paper also looks into the computational issues related to the proposed method’s implementation. A computational procedure is presented to avoid costly re-estimations which would be otherwise required to test the various alternative hypotheses that arise in connection with the problem.
2 Modeling Bilateral Transactions in State Estimation

2.1 Network Modeling

We consider the interconnected power system as composed of two subnetworks: the internal network and the external network. The former is the ISO control area. The external network includes the tie lines to other subsystems. The boundary buses are those buses which are common to both the internal and the external networks.

In order to simplify the description of the proposed method for undisclosed bilateral transaction identification, we use a DC network model to describe the internal system. As it is well known, the so-called DC model is able to provide branch MW flows only; also, flow computation is based on the assumption that all branch resistances are negligible [6]. Those simplifications, however, are not a matter of concern in the current application, since our attention is focused on MW transactions and MW parallel flows. Furthermore, the power systems we are dealing with involve high voltage transmission networks whose branches exhibit high X/R ratios. Under such conditions, the MW flow values obtained from the DC model are seen as reasonably accurate for the level of accuracy expected from the proposed monitoring tool.

If \( N \) is the total number of buses in the internal system including the boundary buses, the basic DC network model equation is given by [6]:

\[
\mathbf{B} \mathbf{\theta} = \mathbf{p}
\]

where \( \mathbf{B} \) is the DC model coefficient matrix [6], \( \mathbf{\theta} \) is the \( N \times 1 \) vector of voltage bus angles and \( \mathbf{p} \) is the \( N \times 1 \) vector of real power bus injections. In the current application, it is convenient to partition matrix \( \mathbf{B} \) and vector \( \mathbf{p} \) according to the definition of internal and boundary buses, leading to the following form for Eq. (1):

\[
\begin{bmatrix}
\mathbf{B}_I \\
\mathbf{B}_B
\end{bmatrix}
\begin{bmatrix}
\mathbf{\theta} \\
\mathbf{p}_B
\end{bmatrix} =
\begin{bmatrix}
\mathbf{p}_I \\
\mathbf{p}_B
\end{bmatrix}
\]

where \( \mathbf{p}_I \) and \( \mathbf{p}_B \) are the vectors of power injections at the strictly internal buses and at the boundary buses, respectively, while \( \mathbf{B}_I \) and \( \mathbf{B}_B \) are the corresponding partitions of matrix \( \mathbf{B} \).
2.2 Bilateral Transactions

Ordinarily, all injections at the boundary buses are announced to the ISO by the neighboring systems. However, in practice this may not always occur. Undisclosed bilateral transactions produce unexpected bus injections at the boundary buses. Therefore, we consider two types of bilateral transactions. Disclosed bilateral transactions are perfectly known by the ISO whose network is affected by the transaction, in the sense that he is aware of both the transacted amount of power and the producer and consumer buses. Therefore, such transactions can be simply modeled as bus power injections, whose signs reflect the type (generator or load) of the corresponding bus.

The amount of power of unreported transactions, on the other hand, is unknown. However, it is conceivable to think that the ISO, based on experience, has a fairly good picture of its control area boundary buses which are exposed to possible bilateral transactions. In this paper, we assume that the ISO can estimate with reasonable accuracy the fraction of the unreported transacted power attributable to each boundary bus. Based on the above arguments, it is possible to uniquely describe a undisclosed transaction $T_i$ using the triplet $T_i = \{s_i, b_i, t_i\}, \ i = 1, \ldots, n_t$ (3)

where $N_B$ is the number of boundary buses in the ISO system, and:

- $s_i = [\sigma^i_1, \sigma^i_2, \ldots, \sigma^i_{N_B}]^T$ is an $N_B \times 1$ vector comprising the production-side participation factors for transaction $i$, which define the proportion of the transacted power injected at each boundary bus;
- $b_i = [\beta^i_1, \beta^i_2, \ldots, \beta^i_{N_B}]^T$ is an $N_B \times 1$ vector comprising the demand-side participation factors for transaction $i$, which define the proportion of the transacted power extracted at each boundary bus, and
- $t_i$ is the (unknown) amount of transacted power for transaction $T_i$.

Notice that, for a given transaction $i$, the $\sigma$ coefficients are zero for the consumer buses and non-negative for the producer buses. Similarly, the $\beta$ coefficients are zero for the producer buses and non-negative for the consumer buses. In addition, both coefficients must satisfy the conditions:

$$\sum_{k=1}^{N_B} \sigma^i_k = 1 \quad \text{and} \quad \sum_{k=1}^{N_B} \beta^i_k = 1$$

The existence of transaction $i$ produces a power injection vector at the boundary nodes given by:

$$p^i_B = (s_i - b_i) t_i \overset{\Delta}{=} m_i t_i$$

(4)

Let $p^0_B$ be the $N_B \times 1$ vector of known external injections at the boundary buses. The boundary bus injection vector considering both its disclosed and
undisclosed components is:

\[ p_B = p_B^0 + \sum_{i=1}^{n_t} m_i t_i \]  \hspace{1cm} (5)

Defining:

\[ M \triangleq \begin{bmatrix} m_1 & m_2 & \ldots & m_{n_t} \end{bmatrix} \]

and

\[ t \triangleq \begin{bmatrix} t_1 & t_2 & \ldots & t_{n_t} \end{bmatrix}^T \]

we can re-write Eq. (5) as

\[ p_B = p_B^0 + Mt \]  \hspace{1cm} (6)

Furthermore, by using Eq. (2) we can write the following equation relating the unreported transactions and the bus voltage angles:

\[ B_B \theta - Mt = p_B^0 \]  \hspace{1cm} (7)

3 Embedding Bilateral Transactions in the State Estimation Formulation

3.1 Extended State Estimation

The ISO control area is monitored through telemetered measurements consisting of certain bus injections and line flows within the internal network. It is assumed that sufficient telemetry is available to render the internal network observable.

We consider the transaction vector \( t \) as a set of new state variables to be estimated. Accordingly, the state vector \( x \) becomes:

\[ x \triangleq \begin{bmatrix} \theta^T : t^T \end{bmatrix}^T, \]  \hspace{1cm} (8)

Using the above definition, the linear measurement model is given by:

\[ z_m = H_m x + \epsilon_m \]  \hspace{1cm} (9)

where \( H_m \triangleq [H_\theta : 0]^T \) and the \( m \times N \) matrix \( H_\theta \) relates the telemetered flow and injection measurements to the bus voltage angles. \( \epsilon_m \) is the \( m \times 1 \) vector of measurement errors, which is assumed to be zero mean and whose (diagonal) covariance matrix is denoted by \( R_m \).
In addition to Eq. (9), state vector components $\theta$ and $t$ must also satisfy Eq. (7). The latter is re-written in terms of the new state vector $x$ as:

$$p_B^0 - H_b x = 0$$

(10)

where $H_b \triangleq [B_B : - M]$.

The state variables are estimated by applying the weighted least-squares method, which minimizes the weighted sum of the squared residuals. The weighting matrix is the inverse of the error covariance matrix $R_m$, and the estimation residuals are defined as:

$$r_m = z_m - H_m x$$

(11)

We formulate the state estimation problem as a constrained optimization problem, where the constraints are given by Eqs. (11) and (10):

Minimize \( \frac{1}{2} r_m^T R_m^{-1} r_m \)

Subject to: \( z_m - H_m x - r_m = 0 \)

\( p_B^0 - H_b x = 0 \)

In practice, the definition of the angular reference $\theta_r = 0$ must also be included as an additional equality constraint to circumvent the singularity of matrix $B$; for simplicity, that constraint is omitted in the following derivations.

### 3.2 A Priori Information

The standard formulation of the state estimation problem does not use any $a$ priori information regarding the state variables. $A$ priori information may be helpful to prevent observability problems in parts of the network where few measurements are available. In this paper, $a$ priori information is modeled directly in the objective function of the optimization problem above, by appending the term:

\[ \frac{1}{2} (x - \bar{x})^T P^{-1} (x - \bar{x}) \]

where $\bar{x} = [\bar{\theta}^T; \bar{t}^T]^T$ and $P = diag\{P_\theta, P_t\}$ are the $a$ priori state vector and its corresponding covariance matrix, respectively. Taking into account the $a$ priori information, the state estimation problem including bilateral transactions becomes:
Minimize \( \frac{1}{2} \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m + \frac{1}{2} (\mathbf{x} - \bar{\mathbf{x}})^T \mathbf{P}^{-1} (\mathbf{x} - \bar{\mathbf{x}}) \)

Subject to:

\[
\begin{align*}
\mathbf{z}_m - \mathbf{H}_m \mathbf{x} - \mathbf{r}_m &= 0 \\
\mathbf{p}_B^0 - \mathbf{H}_b \mathbf{x} &= 0
\end{align*}
\]

In the absence of better information about \textit{a priori} values for \( \theta \), one can assume \( \bar{\theta}_i = 0 \text{ rad} \), \( i = 1, \ldots, N \). To define the covariance matrix \( \mathbf{P}_\theta \), we assume that the \( \bar{\theta}_i \)'s are uncorrelated and uniformly distributed [8] in the interval \([ -\theta_{\text{lim}}, \theta_{\text{lim}} ]\), where \( \theta_{\text{lim}} \) establishes an upper bound for \( \theta \) values under steady-state stable conditions (for instance, \( \theta_{\text{lim}} = \pi/2 \text{ rad} \)). A similar procedure could be adopted for the transacted powers, \( t_i \). In this paper, however, \textit{a priori} values for \( t_i \) are not used, what amounts to making \( \mathbf{P}_t = \rho \mathbf{I} \), where \( \mathbf{I} \) is the identity matrix and \( \rho \to \infty \).

\section*{3.3 The Null Hypothesis}

Since the ISO does not know the amount of the bilateral transactions, the hypothesis is initially made that \textit{no bilateral transactions have occurred}. This is tantamount to saying that the transacted power for every possible unreported transaction is initially assumed zero. We refer to this assumption as the \textit{null hypothesis} \( \mathcal{H}_0 \), and it is simply stated in terms of the state vector \( \mathbf{x} \) as:

\[
\mathcal{H}_0 : \mathbf{H}_t \mathbf{x} = 0
\]  

(12)

where \( \mathbf{H}_t \overset{\Delta}{=} [0: \mathbf{I}] \). The null hypothesis can be considered as an additional set of equality constraints to the state estimation problem, which can thus be re-stated as:

Minimize \( \frac{1}{2} \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m + \frac{1}{2} (\mathbf{x} - \bar{\mathbf{x}})^T \mathbf{P}^{-1} (\mathbf{x} - \bar{\mathbf{x}}) \)

Subject to:

\[
\begin{align*}
\mathbf{z}_m - \mathbf{H}_m \mathbf{x} - \mathbf{r}_m &= 0 \\
\mathbf{p}_B^0 - \mathbf{H}_b \mathbf{x} &= 0 \\
- \mathbf{H}_t \mathbf{x} &= 0
\end{align*}
\]

(13)

The first-order necessary conditions for the optimal solution of Problem (13), i.e. the Karush-Kuhn-Tucker (\textit{KKT}) conditions, are expressed in terms of the
Lagrangian function of the problem:

\[ \mathcal{L} = \frac{1}{2} r_m^T R_m^{-1} r_m + \frac{1}{2} (x - \bar{x})^T P^{-1} (x - \bar{x}) + \lambda_m^T (z_m - H_m x - r_m) + \lambda_b^T (p_B^0 - H_b x) + \lambda_t^T (-H_t x) \]  

(14)

where \( \lambda_m, \lambda_b \) and \( \lambda_t \) are the Lagrange multipliers associated to the measurement, boundary injection and transaction constraints, respectively. The KKT conditions are:

\[ \frac{\partial \mathcal{L}}{\partial r_m} = R_m^{-1} r_m - \lambda_m = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial x} = P^{-1} (x - \bar{x}) - H_m^T \lambda_m - H_b^T \lambda_b - H_t^T \lambda_t = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial z_m} = z_m - H_m x - r_m = 0 \]  

(15)

\[ \frac{\partial \mathcal{L}}{\partial \lambda_b} = p_B^0 - H_b x = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial \lambda_t} = -H_t x = 0 \]

We can use the first and third KKT conditions to eliminate \( r_m \). The reduced set of equations can then be written in matrix form as:

\[
\begin{bmatrix}
-P^{-1} H_m^T & H_b^T & H_t^T \\
H_m & R_m & 0 \\
H_b & 0 & H_t \\
\end{bmatrix}
\begin{bmatrix}
x \\
\lambda_m \\
\lambda_b \\
\lambda_t \\
\end{bmatrix}
= 
\begin{bmatrix}
-P^{-1} \bar{x} \\
z_m \\
p_B^0 \\
0 \\
\end{bmatrix}
\]  

(16)

4 Hypothesis Testing for Transaction Identification

4.1 Alternative Hypotheses

The purpose of hypothesis testing is to establish whether the experimental data, i.e. the measurements \( z_m \in \mathbb{R}^m \), support the null hypothesis or some other hypothesis about unreported transactions. Considering the existence of
such transactions, there are $2^{n_t}$ possible combinations of them. Each of those combinations corresponds to a different hypothesis. Since $2^{n_t}$ can be a very large number, it would not be practical to go through every possible combination to identify the one which is actually occurring. Nevertheless, we consider that approach in this section to outline the proposed method. A more practical approach to perform hypothesis testing for a large number of hypotheses is presented in the next section.

Let $\mathcal{H}_i$ be the hypothesis that combination $i$ of undisclosed transactions has occurred. For $i = 0$, we clearly have the null hypothesis $\mathcal{H}_0$. Each of the remaining hypotheses $\mathcal{H}_i$, $i > 0$, refers to the occurrence of at least one unreported transaction, and is called an alternative hypothesis.

### 4.2 Testing Alternative Hypotheses through Re-estimation

Whenever the available data does not support the null hypothesis, one can expect that some of the Lagrangian multipliers associated with equality constraints (10) or (12) will take very large values. It is clear from the first KKT condition in Eq. (15) that the Lagrange multipliers exhibit a behavior similar to the estimation residuals. In fact, Lagrange multipliers can be statistically characterized, normalized and subsequently employed for anomaly detection and identification, in a similar manner as the residuals are [5],[9].

A false null hypothesis can thus be detected by comparing the maximum normalized Lagrange multiplier with a given threshold $\gamma$. This threshold is determined by a specified level $\alpha$ of false alarm probability [3] (for instance, under the Gaussian distribution assumption a threshold $\gamma = 3.0$ corresponds to $\alpha < 1\%$). In case the maximum normalized Lagrange multiplier exceeds $\gamma$, a possible strategy to identify unreported transactions could be based on the systematic relaxation of individual constraints in Eq. (12), thereby generating multiple alternative hypotheses. For each of them, state estimation would again be performed and the detection test using normalized Lagrange multipliers would be re-applied. This procedure goes on until some alternative hypothesis is singled out as consistent with the measurement data. In this scheme, hypothesis relaxation is implemented by simply replacing the zero diagonal entry on the same row of the relaxed constraint in Eq. (16) by a large positive number. This is equivalent to assigning a large variance to the constraint being relaxed.

Provided that enough measurement redundancy is available, the procedure outlined above is capable of identifying the combination of transactions that is actually occurring. However, it becomes impractical if a significant number of transactions is considered, since the required number of runs of the state
estimator would demand a large computational effort. Fortunately, this can be avoided through the application of Bayes’ theorem, as shown in the next section.

5 Hypothesis Testing via Bayes’ Theorem

5.1 Statistical Background

In this section, our objective is to determine which hypothesis is best supported by the current set of measurements. Bayes’ theorem is an adequate tool to perform this task, since it is able to compute the \textit{a posteriori} probability of a given hypothesis on the basis of: (a) the probability density function (pdf) of the measurements assuming the hypothesis is true, and (b) the \textit{a priori} probability that the hypothesis holds. The former can be determined from the probability distribution of measurement errors, as discussed in the sequel, while the \textit{a priori} probabilities should be provided by the user. If no reliable information about the likelihood of the hypotheses is available, they can be assumed to be equally likely.

Let $P(\mathcal{H}_i)$ be the \textit{a priori} probability of hypothesis $i$ and $P(\mathcal{H}_i|z)$ be the \textit{a posteriori} probability. The relevant form of Bayes’ theorem states that [8]

$$
P(\mathcal{H}_i|z) = \frac{f(z|\mathcal{H}_i)P(\mathcal{H}_i)}{\sum_j^N f(z|\mathcal{H}_j)P(\mathcal{H}_j)}$$

(17)

where $f(z|\mathcal{H}_i)$ is the conditional probability density function of $z$ given that $\mathcal{H}_i$ is true.

Since the \textit{a priori} probabilities are given, the \textit{a posteriori} probability $P(\mathcal{H}_i|z)$ can be determined provided that conditional probability density functions $f(z|\mathcal{H}_i)$ are available.

5.2 Computing the Conditional Pdf

For convenience, we redefine:

$$
\begin{align*}
\mathbf{z} & \triangleq \begin{bmatrix} z_m \\ p_B^0 \\ 0 \end{bmatrix} \\
\mathbf{H} & \triangleq \begin{bmatrix} H_m \\ H_b \\ H_t \end{bmatrix} \\
\epsilon & \triangleq \begin{bmatrix} \epsilon_m \\ 0 \\ 0 \end{bmatrix}
\end{align*}
$$

(18)
and \( R_0 = \text{diag}\{R_m, \sigma_b^2 I_{N_B}, \sigma_t^2 I_{n_t}\} \), where we have replaced the null diagonal matrices in Eq. (16) by matrices \( \varepsilon I_{N_B} \) and \( \varepsilon I_{n_t} \). Scalars \( \sigma_b^2 \) and \( \sigma_t^2 \) are a small positive numbers and \( I_{N_B} \), \( I_{n_t} \) are the \( N_B \times N_B \) and \( n_t \times n_t \) identity matrices, respectively.

The measurements are related to the system static state vector through the measurement equation

\[
z = Hx + \epsilon
\]

where \( x \in \mathbb{R}^n \) and \( \epsilon \) is a vector of measurement errors.

The calculation of \( f(z|H_i) \) is required to determine the hypotheses’ conditional probabilities. This is readily implemented if \( x \) and \( \epsilon \) are assumed to be Gaussian random variables. With [8]

\[
f_x(x) = \left[ (2\pi)^n |P| \right]^{-1/2} e^{-\frac{1}{2}(x-x)^T P^{-1}(x-x)}
\]

and

\[
f_\epsilon(\epsilon|H_i) = \left[ (2\pi)^m |R_i| \right]^{-1/2} e^{-\frac{1}{2}\epsilon^TR_i^{-1}\epsilon}
\]

then \( f(z|H_i) \) is Gaussian with pdf [8]

\[
f(z|H_i) = \left[ (2\pi)^m |\Omega_i| \right]^{-1/2} e^{-\frac{1}{2}(z-Hx)^T \Omega_i^{-1}(z-Hx)}
\]

where \( \Omega_i \) is the error covariance matrix for hypothesis \( H_i \) and

\[
\Omega_i = R_i + HPH^T
\]

5.3 Applying Bayes’ Theorem to Transaction Identification

Hypothesis \( H_i, i \neq 0 \), establishes that certain unreported transactions are in fact nonzero. Suppose that \( H_i \) comprises \( k \) of such transactions. Let \( i_1, \ldots, i_k \) denote the diagonal entries in \( R_i \) that are set equal to a large positive number in order to release the zero assumption on the transaction value. Thus

\[
R_i = R_0 + \zeta E_i E_i^T
\]

where \( \zeta \) is the large positive number, \( E_i = \begin{bmatrix} e_{i_1} & \cdots & e_{i_k} \end{bmatrix} \) and \( e_j \) is the \( j \)-th column of the identity matrix.

In order to evaluate the conditional pdf, we see from Eq. (19) that both \( |\Omega_i| \) and \( (z-Hx)^T \Omega_i^{-1}(z-Hx) \) must be computed. Starting with the latter, we apply the Sherman-Morrison-Woodbury formula [10] to obtain \( \Omega_i^{-1} \) from Eq. (20). For a matrix \( P \) with relatively large entries, that yields:

\[
\Omega_i^{-1} = R_i^{-1} - R_i^{-1}H \left( H^T R_i^{-1}H \right)^{-1} H^T R_i^{-1}
\]
Using this result and defining \( \hat{x}_i \) as the state estimate under hypothesis \( H_i \) and \( r^i = z - H \hat{x}_i \) as the corresponding estimation residual, we obtain:

\[
(z - H \hat{x})^T \Omega_i^{-1} (z - H \hat{x}) = (z - H \hat{x})^T R_i^{-1} r^i
\]  

(23)

We now turn our attention to \( |\Omega_i| \). Direct calculation of this determinant for each hypothesis can be a computationally intensive task if the number of hypotheses is large. We will use a determinant identity in order to reduce the computational burden. From Eqs. (20) and (21), we rewrite \( \Omega_i \) as

\[
\Omega_i = \Omega_0 \left( I_m + \zeta \Omega_0^{-1} E_i E_i^T \right)
\]

so that [10]

\[
|\Omega_i| = |\Omega_0| \left| I_m + \zeta \Omega_0^{-1} E_i E_i^T \right| = |\Omega_0| \left| I_k + \zeta E_i^T \Omega_0^{-1} E_i \right|
\]

(24)

Since \( k \ll m \), the expression on the far r.h.s. of Eq. (24) provides a much more efficient means to compute \( |\Omega_i| \) than its straightforward calculation from Eq(20). In addition, we show in the Appendix how the product \( E_i^T \Omega_0^{-1} E_i \) and the residual vector \( r^i \) of Eq. (23) can be efficiently computed by using the information matrix triangular factors available from the base case state estimation solution. Essentially, the developments in the Appendix show that both \( |\Omega_i| \) and \( r^i \) corresponding to hypothesis \( i \), \( i \neq 0 \), can be obtained by imposing modifications on \( |\Omega_0| \) and \( r^0 \) computed for the base case. That is to say, no further runs of the state estimation other than that corresponding to the base case are required to compute the \textit{a posteriori} hypothesis probabilities given by Eq. (17).

6 Conditions for Transaction Identification

For a given power network containing \( N_B \) boundary buses, there is a maximum number of unreported transactions that can be identified through the proposed methodology. This limit is strongly related to observability issues, i.e. to the rank of the augmented observation matrix \( H \) defined in Eq. (18).

Assuming that the network is observable in the conventional nodal state sense [11], it can be shown that the number \( n_t \) of identifiable transactions is subject to an upper bound that depends on the number of boundary buses. In fact, it is possible to establish the following necessary condition for transaction identification:

\[
n_t \leq N_B - 1
\]

(25)

The proof of the above result is not included in this paper due to space limitations, but can be found in reference [12].
7 Simulation Results

The proposed method for transaction identification has been tested on several power networks, including the IEEE 14-bus and 118-bus test systems. The results presented in this paper refer to the 118-bus network, whose one-line diagram is shown in Fig. 1 [13]. The system loading is based on the data in [14], except for the boundary buses, whose injections are considered zero. To accommodate this, the original loads at each boundary bus is dispersed among the adjacent buses.

The metering scheme for the test system comprises 289 measurements: power flows are measured at one end of all network branches and bus power injections are monitored at all buses but the boundary buses. Measurements are considered noisy, by superposing randomly generated errors on values obtained from a load flow study. A priori information has been assigned only to bus voltage angles ($\bar{\theta}_i = 0$ for $i = 1, \ldots, N$).

Throughout this section, it is assumed that the 118-bus system corresponds to the ISO control area, and that there are 8 boundaries with neighboring systems. The boundary buses are: buses 8 and 19, which belong to the northwestern part of the system; bus 27, in the Southwest; bus 55, in the Northeast; bus 62, in the Eastern region; bus 106, in the Southeast, and buses 24 and 69, which pertain to the Central region. To facilitate visualization, the boundary bus numbers are shown in black boxes in Fig. 1.

Table 1 describes the transactions considered in this paper. For simplicity, we consider that each transaction involves a single seller and a single buyer. Therefore, if $p$ and $c$ are the producer and consumer buses, respectively, then $\sigma_p = 1$ and $\sigma_k = 0$ for all $k \neq p$. Similarly, $\beta_c = 1$ and $\beta_k = 0$ for all $k \neq c$.

Table 1
Transaction Data

<table>
<thead>
<tr>
<th>Trans.</th>
<th>p bus</th>
<th>c bus</th>
<th>Value (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tr1</td>
<td>8 (NW)</td>
<td>27 (SW)</td>
<td>0.23</td>
</tr>
<tr>
<td>Tr2</td>
<td>55 (NE)</td>
<td>19 (NW)</td>
<td>0.77</td>
</tr>
<tr>
<td>Tr3</td>
<td>62 (E)</td>
<td>55 (NE)</td>
<td>0.63</td>
</tr>
<tr>
<td>Tr4</td>
<td>106 (SE)</td>
<td>24 (C)</td>
<td>0.16</td>
</tr>
<tr>
<td>Tr5</td>
<td>106 (SE)</td>
<td>69 (C)</td>
<td>0.36</td>
</tr>
<tr>
<td>Tr6</td>
<td>8 (NW)</td>
<td>69 (C)</td>
<td>0.42</td>
</tr>
<tr>
<td>Tr7</td>
<td>19 (NW)</td>
<td>24 (C)</td>
<td>0.99</td>
</tr>
<tr>
<td>Tr8</td>
<td>27 (SW)</td>
<td>62 (E)</td>
<td>0.57</td>
</tr>
</tbody>
</table>
The simulated cases to be discussed next differ from each other on the number of possible transactions and on the combinations of undisclosed transactions actually occurring in the system.

### 7.1 Four Possible Transactions

In case A, only transactions \( Tr1 \) through \( Tr4 \) of Table 1 can possibly occur. Under this assumption, three cases are simulated:

- **Case A.1**: Only transaction \( Tr1 \) (Combination 1000) is in fact taking place;
- **Case A.2**: Transactions \( Tr1 \) and \( Tr2 \) are taking place, transactions \( Tr3 \) and \( Tr4 \) are inactive (Combination 1100);
- **Case A.3**: Transactions \( Tr1 \), \( Tr2 \) and \( Tr4 \) are occurring, transaction \( Tr3 \) is inactive (Combination 1101).

The results for the above cases are summarized in the first three rows of Table 2 and in Table 3. Results in Table 2 show that the maximum normalized
Table 2
Lagrange Multipliers and *A Posteriori* Probabilities

| Case | $(\lambda^0_N)_{\text{max}}$ (*) | Max. $P(\mathcal{H}_i|z)$ | $(\lambda_i^f)_{\text{max}}$ (†) |
|------|--------------------------------|-------------------------|--------------------------|
|      | Value                        | Comb.                   |                          |
| A.1  | 132.33                       | 0.9921                  | 1000                     | 1.80                     |
| A.2  | 457.34                       | 0.9970                  | 1100                     | 2.14                     |
| A.3  | 457.28                       | 0.9993                  | 1101                     | 2.24                     |
| B.1  | 457.28                       | 0.9923                  | 110100                   | 2.24                     |
| B.2  | 465.14                       | 0.9970                  | 111100                   | 2.21                     |
| C.1  | 465.14                       | 0.9874                  | 11110000                 | 2.21                     |
| C.2  | 459.20                       | 0.9924                  | 11111100                 | 2.01                     |

(*) Max. normalized Lagrange multiplier for hypothesis $\mathcal{H}_0$

(†) Max. normalized Lagrange multiplier for selected $\mathcal{H}_i$

Lagrange multiplier computed under the null hypothesis takes a large value for all three cases. This implies that the existing data do not support the null hypothesis, which is therefore rejected. The *a posteriori* probabilities for all possible alternative hypotheses are then computed. The center columns of Table 2 show the largest *a posteriori* probability assigned to a single alternative hypothesis (out of the 15 possible candidates) and the corresponding combination of transactions. The probability value shown approaches 1.0 (typically, the second largest probability value is less than $1 \times 10^{-2}$). As one can readily verify, the selected alternative hypothesis coincides with the combination of transactions simulated as active, for each of the cases A.1, A.2 and A.3.

After identifying the transaction combination best supported by the available data, the estimation of the transaction amounts is performed. This is done by relaxing the original constraints of the null hypothesis corresponding to the transactions now found to be active. The last column of Table 2 indicates that, under the selected alternative hypothesis, the maximum normalized Lagrange multiplier takes a value below the usual threshold of 3.0. The estimated active transaction values shown in Table 3 are close to the simulated ones shown in Table 1.

7.2 Six Possible Transactions

Cases B.1 and B.2 assume that transactions $Tr1$ through $Tr6$ of Table 1 may possibly occur. Case B.1 considers that, out of the six possible transactions, only $Tr1$, $Tr2$ and $Tr4$ are actually active, that is, it corresponds to combi-
Table 3
Transaction Estimation - Case A

<table>
<thead>
<tr>
<th>Transaction</th>
<th>A.1</th>
<th>A.2</th>
<th>A.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tr1</td>
<td>0.2295</td>
<td>0.2294</td>
<td>0.2295</td>
</tr>
<tr>
<td>Tr2</td>
<td>0.0</td>
<td>0.7703</td>
<td>0.7703</td>
</tr>
<tr>
<td>Tr3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Tr4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1597</td>
</tr>
</tbody>
</table>

In case B.2, the active transactions are Tr1 through Tr4, leading to the combination 111100.

The corresponding results are shown in Table 2 and in the first two columns of Table 4. Again, one can notice from Table 2 that the proposed method is able to single out the right alternative hypotheses, with a \textit{a posteriori} probability close to one. The correctness of those results is confirmed by the low values of the normalized Lagrange multipliers obtained after the right set of active transaction constraints is taken into account. The computed estimates for the transacted amounts, presented in Table 4, approach the simulated values of Table 1.

7.3 Eight Possible Transactions

We finally assume that all eight transactions can possibly occur. Out of those, case C.1 considers the first four transactions as the only actually active (combination 11110000), whereas case C.2 extends the active set to the first six transactions (combination 11111100). Results for both cases also appear in Tables 2 and 4, as referred. Basically, the same remarks made for the previous two subsections apply, namely, the hypothesis testing procedure is effective in selecting the proper alternative hypothesis, which is corroborated by both the values of the corresponding normalized Lagrange multipliers and the final estimated values for the active transactions.

Note that in all studied cases the number of \textit{active} transactions satisfy the necessary condition (25) for transaction identification.

8 Conclusions

Under deregulated electricity markets, an ISO is usually informed of all bilateral transactions affecting the power flows within its control area, but this may
Table 4
Transaction Estimation - Cases B and C

<table>
<thead>
<tr>
<th>Transaction</th>
<th>B.1</th>
<th>B.2</th>
<th>C.1</th>
<th>C.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tr1</td>
<td>0.2295</td>
<td>0.2295</td>
<td>0.2295</td>
<td>0.2303</td>
</tr>
<tr>
<td>Tr2</td>
<td>0.7703</td>
<td>0.7708</td>
<td>0.7708</td>
<td>0.7709</td>
</tr>
<tr>
<td>Tr3</td>
<td>0.0</td>
<td>0.6309</td>
<td>0.6309</td>
<td>0.6309</td>
</tr>
<tr>
<td>Tr4</td>
<td>0.1598</td>
<td>0.1598</td>
<td>0.1598</td>
<td>0.1587</td>
</tr>
<tr>
<td>Tr5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.3614</td>
</tr>
<tr>
<td>Tr6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.4179</td>
</tr>
<tr>
<td>Tr7</td>
<td>-</td>
<td>-</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Tr8</td>
<td>-</td>
<td>-</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

not always be the case. The need then arises to extend the existing monitoring tools available to the ISO in order to allow the identification of undisclosed bilateral transactions.

For that purpose, we have presented in this paper an extension of power system state estimation where transactions are treated as state variables and unreported transaction identification is carried out through a hypothesis testing method based on Bayesian estimation. This approach provides a probability value for each possible hypothesis involving the transactions that may be simultaneously occurring across the boundaries of the control area. Computational techniques to determine the hypothesis probabilities without requiring multiple runs of the state estimator have been developed and are also described in detail.

In order to test the proposed method, the IEEE 118-bus system has been used to simulate a control area subject to several simultaneous unreported transactions. The results show that the method is successful in assigning the highest probability to the combination of transactions actually occurring. Following the identification stage, the method also provides good estimates for the amount of power corresponding to each transaction.
Appendix

This appendix addresses computational issues related to the computation of both $r^i$ and $|\Omega_i|$ that appear on Eqs. (23) and (24), respectively. The latter establishes that

$$|\Omega| / |\Omega_0| = \left| 1 + \zeta E^T_0 \Omega_0^{-1} E_i \right| \quad (1)$$

Making $i = 0$ in Eq. (22) and substituting the result into the matrix product at the r.h.s. of the previous equation, we get

$$E^T_i \Omega^{-1}_0 E_i = \tilde{R}_i^{-1} - \tilde{R}_i^{-1} \tilde{H}_i \left( H^T R_0^{-1} H \right)^{-1} \tilde{H}_i^T \tilde{R}_i^{-1}$$

where

$$\tilde{H}_i^T = \begin{bmatrix} h_{i1} \\ \vdots \\ h_{ik} \end{bmatrix} \quad \text{and} \quad \tilde{R}_i = \begin{bmatrix} \sigma^2_{i1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma^2_{ik} \end{bmatrix} = \sigma^2_i I_k$$

Furthermore, let $U$ be the upper triangular factor available from the state estimation solution of the base case, that is, $H^T R_0^{-1} H = U^T D U$. Therefore:

$$E^T_i \Omega^{-1}_0 E_i = \tilde{R}_i^{-1} - \tilde{R}_i^{-1} \tilde{H}_i \left( H^T R_0^{-1} H \right)^{-1} \tilde{H}_i^T \tilde{R}_i^{-1} \quad (2)$$

where the columns of $\tilde{H}_i^T = U^{-T} \tilde{H}_i^T$ can be quickly computed by fast forward substitution, since the columns of $\tilde{H}_i^T$ are sparse vectors [15]. By using Eq. (2.2) into Eq. (1.1), $|\Omega_i|$ can be efficiently computed.

We next consider the calculation of $r^i = z - H \hat{x}_i$. The state estimate, $\hat{x}_i$, must satisfy the normal equations $H^T R_i^{-1} H \hat{x}_i = H^T R_i^{-1} z$. Since $R_i = R_0 + \zeta E_i E^T_i$, we have

$$R_i^{-1} = R_0^{-1} - E_i \Gamma E^T_i$$

where $\Gamma$ is a diagonal matrix whose diagonal entries are $\gamma_j = (\zeta / \sigma^2_{ij}) / (\zeta + \sigma^2_{ij})$ and $\sigma^2_{ij}$ is the $i - j$th diagonal entry of $R_0$. We write the normal equation for $\hat{x}_i$ as

$$H^T \left( R_0^{-1} - E_i \Gamma E^T_i \right) H \hat{x}_i = H^T \left( R_0^{-1} - E_i \Gamma E^T_i \right) z$$

or

$$(C_0 - \tilde{H}_i^T \Gamma \tilde{H}_i) \hat{x}_i = H^T R_0^{-1} z - \tilde{H}_i^T \Gamma E^T_i z$$

where $C_0 = H^T R_0^{-1} H$. We can thus write $\hat{x}_i$ as

$$\hat{x}_i = \left( C_0 - \tilde{H}_i^T \Gamma \tilde{H}_i \right)^{-1} \left( H^T R_0^{-1} - \tilde{H}_i^T \Gamma E^T_i \right) z$$

By applying the Sherman-Morrison-Woodbury formula to evaluate the inverse on the r.h.s., we have

$$(C_0 - \tilde{H}_i^T \Gamma \tilde{H}_i)^{-1} = C_0^{-1} + W_i \left( \Gamma^{-1} - \tilde{H}_i W_i \right)^{-1} W_i^T$$
where $W_i = C_0^{-1} \tilde{H}_i^T$. Using this result in the expression for $\tilde{x}_i$ and after some manipulations, we get

$$\tilde{x}_i = \tilde{x}_0 - W_i \left( \Gamma^{-1} - \tilde{H}_i W_i \right)^{-1} \tilde{r}_i \tag{3}$$

where $\tilde{r}_i = E_i^T z - \tilde{H}_i \tilde{x}_0$. We can write $W_i$ as $W_i = U^{-1} D^{-1} U^{-T} \tilde{H}_i^T$, so that equation (.3) can be re-written as

$$\tilde{x}_i = \tilde{x}_0 - U^{-1} D^{-1} \tilde{H}_i^T \left( \Gamma^{-1} - \tilde{H}_i D^{-1} \tilde{H}_i^T \right)^{-1} \tilde{r}_i$$

The matrix expression enclosed by the parentheses on the r.h.s. is a small dense matrix and therefore to perform its product by $\tilde{r}_i$ is an easy task. Once we have calculated $\tilde{H}_i^T = U^{-T} \tilde{H}_i^T$ by fast forward substitution we need to do one small dense vector matrix multiply, one division by a diagonal and one sparse back substitution to compute the term subtracted from $\tilde{x}_0$ on the r.h.s.. When the computations are organized in the manner described above, the major computational task is one sparse back substitution per hypothesis test.

References


